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AND
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Submitted in partial fulfillment of
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MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
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ABSTRACT

A theoretical analysis was made of the torque and the space harmonics of magnetomotive force produced by a square wave impressed voltage. The results of this analysis were then verified experimentally.

With the motor wye connected without a neutral, a comparison of the characteristics produced by the square voltage wave to those produced by a sinusoidal voltage wave shows little difference. With the neutral connected there was a decided increase in losses, noise and vibration.

The writers wish to express their appreciation for the assistance and encouragement given them by Professor Orval H. Polk of the U. S. Naval Postgraduate School in this investigation.

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1. Introduction.

The squirrel cage induction motor has long been known for its ruggedness and reliability when operated on sinusoidal voltages. In response to a suggestion by the Bureau of Ships, Navy Department, Washington, D.C., an investigation of the effect of a square wave impressed voltage on a squirrel cage induction motor was undertaken at the U. S. Naval Postgraduate School, Monterey, California. The investigation was commenced in October 1958.

The non-sinusoidal voltage used was an approximation of a square wave such as that obtained from a static inverter. A static inverter is a device which is capable of transferring energy from a direct current source to an alternating current load. The voltage used was obtained from a harmonic generator set.

The analysis of the performance of induction motors operating on sinusoidal voltages is well known.[1] The purpose of this investigation was to determine the effect of a square voltage wave upon the theoretical torque-speed characteristics and efficiencies of a squirrel cage induction motor and to compare these characteristics with those obtained by operating the motor on sinusoidal voltages.

The investigation was divided into two parts. The first part was a theoretical analysis of the torque producing magneto-motive forces set up by a three phase square wave impressed voltage.

The second part was to obtain speed-torque curves with both sinusoidal and square wave impressed voltages so that a comparison of the speed-torque characteristics could be made.

[1] Alger, The Nature of Polyphase Induction Machines, John Wiley & Sons, 1951.

2. Theoretical Analysis of Magnetomotive Force Waves.

The torque producing phenomena of a three phase induction motor run by impressing a square voltage wave per phase are determined by resolving a square wave into a series of sinusoidal waves by Fourier analysis.[1] For a three phase motor the square phase voltages must be displaced from each other by $\frac{2\pi}{3}$ radians.

The Fourier analysis of the phase voltages Ea, Eb and Ec gives:

$$E_a = \frac{4}{\pi}\left\{E\sin\omega t + \frac{E}{3}\sin 3\omega t + \cdots + \frac{E}{n}\sin n\omega t + \cdots\right\} \quad (1)$$

$$E_b = \frac{4}{\pi}\left\{E\sin\left(\omega t - \frac{2\pi}{3}\right) + \frac{E}{3}\sin 3\left(\omega t - \frac{2\pi}{3}\right) + \cdots + \frac{E}{n}\sin n\left(\omega t - \frac{2\pi}{3}\right) + \cdots\right\} \quad (2)$$

$$E_c = \frac{4}{\pi}\left\{E\sin\left(\omega t + \frac{2\pi}{3}\right) + \frac{E}{3}\sin 3\left(\omega t + \frac{2\pi}{3}\right) + \cdots + \frac{E}{n}\sin n\left(\omega t + \frac{2\pi}{3}\right) + \cdots\right\} \quad (3)$$

where E is the maximum voltage of the square wave, and n is an odd integer. Fig. 1 shows the phasor diagram of the voltages through the seventh harmonic.

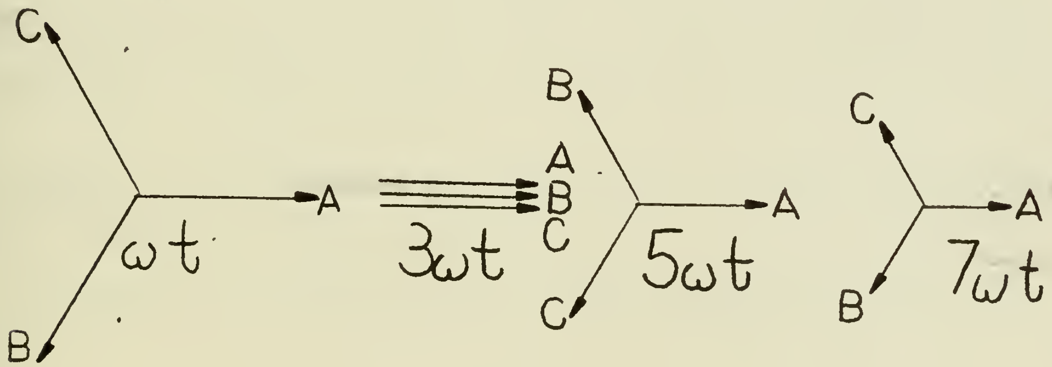

Fig. 1 Voltage Phasor Diagrams

[1]Kerchner and Corcoran, Alternating Current Circuits, Second Edition, John Wiley & Sons, 1944, p 141.

The third and odd multiples of the third harmonic line to neutral voltages are in phase.[1,2] Since all 3n harmonics are in phase there will be no 3n harmonics in the line to line voltages. These 3n harmonic phase voltages impressed on a balanced wye load will produce no 3n harmonic current unless a return path is provided through a neutral connection.

Applying these voltages to a three phase wye connected motor with the neutral connected, each harmonic voltage En will produce a current I_n in the motor windings. These currents are given by:

$$I_a = I_1 \sin(\omega t - \alpha) + I_3 \sin(3\omega t - \beta) + \cdots + I_n \sin(n\omega t - \nu) \tag{4}$$

$$I_b = I_1 \sin\left(\omega t - \frac{2\pi}{3} - \alpha\right) + I_3 \sin\left[3\left(\omega t - \frac{2\pi}{3}\right) - \beta\right] + \cdots + I_n \sin\left[n\left(\omega t - \frac{2\pi}{3}\right) - \nu\right] + \cdots \tag{5}$$

$$I_c = I_1 \sin\left(\omega t + \frac{2\pi}{3} - \alpha\right) + I_3 \sin\left[3\left(\omega t + \frac{2\pi}{3}\right) - \beta\right] + \cdots + I_n \sin\left[n\left(\omega t + \frac{2\pi}{3}\right) - \nu\right] + \cdots \tag{6}$$

where $\alpha, \beta, \cdots \nu$ are the angles by which the currents lag the voltages.

To determine the mmf waves that are produced by these currents it is necessary to find the space distribution of current around the stator.

[1] Kerchner and Corcoran, Alternating Current Circuits, Second Edition, John Wiley & Sons, 1944, pp 190-192.

[2] Fitzgerald and Kingsley, Electric Machinery, McGraw Hill, 1952, pp 256-259.

Assume a direct current applied to a double layer fractional pitch winding with one coil per pole per phase. Further assume the coil sides to have zero width. Let p be the coil pitch and γ be the angular displacement around the stator in radians. Therefore, a plot of ampere turns versus γ for one phase appears as in Fig. 2.

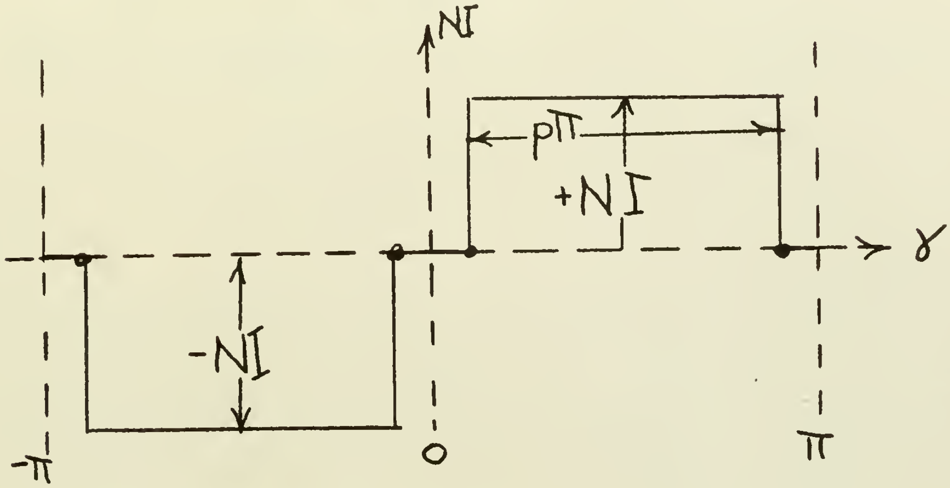

Fig. 2 Ampere Turns versus Angular Displacement

This wave may be resolved into a series of sinusoidal waves. Since $F(\gamma) = -F(-\gamma)$, the cosine terms will drop out and the series appear as:

$$F(\gamma) = \sum_{k=0}^{\infty} a_k \sin k\gamma$$

where

$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} F(\gamma) \sin k\gamma \, d\gamma$$

This is then broken up into two integrals:

$$a_k = \frac{1}{\pi}\left\{ \int_{-\frac{\pi}{2}-\frac{p\pi}{2}}^{-\frac{\pi}{2}+\frac{p\pi}{2}} -NI \sin k\gamma \, d\gamma + \int_{\frac{\pi}{2}-\frac{p\pi}{2}}^{\frac{\pi}{2}+\frac{p\pi}{2}} NI \sin k\gamma \, d\gamma \right.$$

Integrating and substituting the limits:

$$a_k = \frac{4NI}{\pi k} \sin \frac{kp\pi}{2}$$ for k = odd integer

and

$$a_k = 0$$ for k = even integer

The term $\sin \frac{kp\pi}{2}$ is the pitch factor K_{pk} so that the mmf wave is:

$$F(\gamma) = \frac{4NI}{\pi}\left\{ \frac{K_{p1}}{1} \sin\gamma + \frac{K_{p3}}{3} \sin 3\gamma + \cdots + \frac{K_{pk}}{k} \sin k\gamma + \cdots \right\}$$

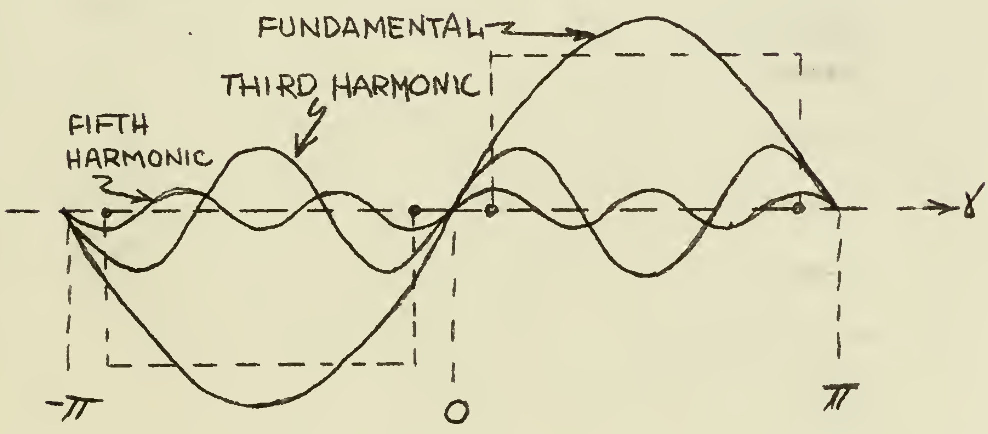

Fig. 3 Harmonics of Flux Waves

Fig. 3 shows the first three harmonics obtained in this way. For a three phase machine the other two windings will be displaced from this winding by $-\frac{2\pi}{3}$ and $\frac{2\pi}{3}$ electrical radians.

Applying the same analysis to the other windings gives the mmf distribution for phases a, b and c as:

$$F(\gamma)_a = \frac{4NI}{\pi}\left\{\frac{K_{p1}}{1}\sin\gamma + \frac{K_{p3}}{3}\sin 3\gamma + \cdots + \frac{K_{pk}}{k}\sin k\gamma + \cdots\right\} \tag{8}$$

$$F(\gamma)_b = \frac{4NI}{\pi}\left\{\frac{K_{p1}}{1}\sin\left(\gamma-\frac{2\pi}{3}\right) + \frac{K_{p3}}{3}\sin 3\left(\gamma-\frac{2\pi}{3}\right) + \cdots + \frac{K_{pk}}{k}\sin k\left(\gamma-\frac{2\pi}{3}\right) + \cdots\right\} \tag{9}$$

$$F(\gamma)_c = \frac{4NI}{\pi}\left\{\frac{K_{p1}}{1}\sin\left(\gamma+\frac{2\pi}{3}\right) + \frac{K_{p3}}{3}\sin 3\left(\gamma+\frac{2\pi}{3}\right) + \cdots + \frac{K_{pk}}{k}\sin k\left(\gamma+\frac{2\pi}{3}\right) + \cdots\right\} \tag{10}$$

For a distributed winding the mmf distribution for the fundamental and third harmonic of one phase appears as in Fig. 4. These sinusoidal mmf waves are added to give the resultant phase mmf for each harmonic.

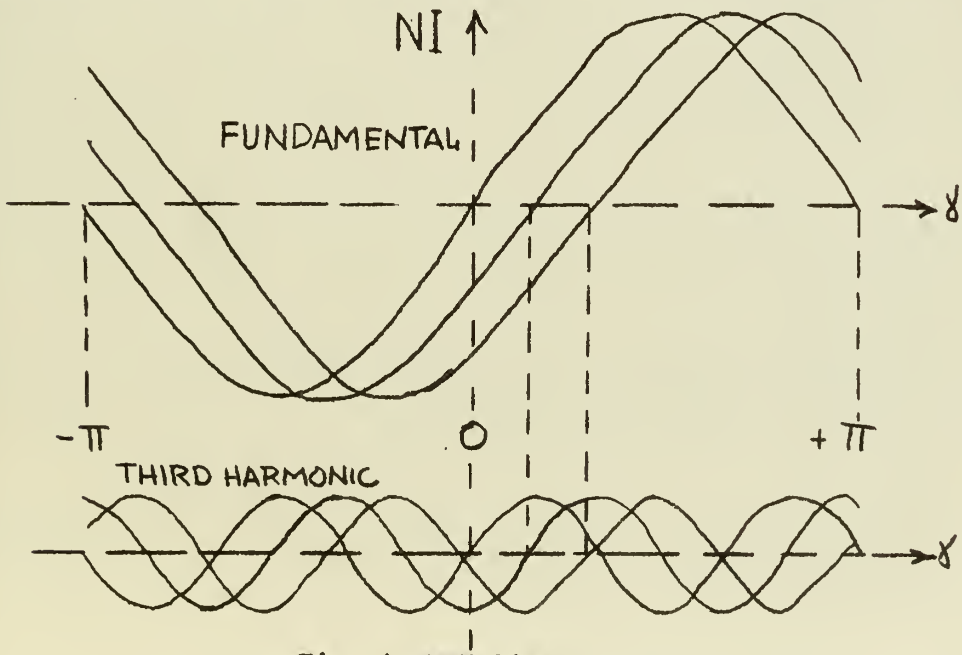

Fig. 4 MMF Distribution

The distribution factor[1] $K_{dk} = \dfrac{\sin \frac{\pi k}{m}}{s \sin \frac{\pi k}{sm}}$ where s is the number of slots per pole per phase and m is the number of phases external to the machine. The space distribution of the mmf waves around the stator is:

$$F(\gamma)_a = \frac{4NI}{\pi}\left\{\frac{K_{p1}K_{d1}}{1}\sin\gamma + \frac{K_{p3}K_{d3}}{3}\sin 3\gamma + \frac{K_{pk}K_{dk}}{k}\sin k\gamma + \cdots\right\} \quad (11)$$

$$F(\gamma)_b = \frac{4NI}{\pi}\left\{\frac{K_{p1}K_{d1}}{1}\sin\left(\gamma-\frac{2\pi}{3}\right) + \frac{K_{p3}K_{d3}}{3}\sin 3\left(\gamma-\frac{2\pi}{3}\right) + \cdots \right.$$
$$\left. \cdots + \frac{K_{pk}K_{dk}}{k}\sin k\left(\gamma-\frac{2\pi}{3}\right) + \cdots\right\} \quad (12)$$

$$F(\gamma)_c = \frac{4NI}{\pi}\left\{\frac{K_{p1}K_{d1}}{1}\sin\left(\gamma+\frac{2\pi}{3}\right) + \frac{K_{p3}K_{d3}}{3}\sin 3\left(\gamma+\frac{2\pi}{3}\right) + \cdots \right.$$
$$\left. \cdots + \frac{K_{pk}K_{dk}}{k}\sin k\left(\gamma+\frac{2\pi}{3}\right) + \cdots\right\} \quad (13)$$

If the square wave alternating currents given by equations (4), (5) and (6) replace the direct current in the above equations, the general mmf expressions for the n^{th} current harmonic and the k^{th} space harmonic become:

$$F(\gamma,t)_a = \frac{4N}{\pi}\,\frac{I_n K_{pk} K_{dk}}{k}\left[\sin(n\omega t - \nu)\right](\sin k\gamma) \quad (14)$$

$$F(\gamma,t)_b = \frac{4N}{\pi}\,\frac{I_n K_{pk} K_{dk}}{k}\left[\sin\left(n\omega t - \frac{n2\pi}{3} - \nu\right)\right]\left[\sin k\left(\gamma-\frac{2\pi}{3}\right)\right] \quad (15)$$

$$F(\gamma,t)_c = \frac{4N}{\pi}\,\frac{I_n K_{pk} K_{dk}}{k}\left[\sin\left(n\omega t + \frac{n2\pi}{3} - \nu\right)\right]\left[\sin k\left(\gamma+\frac{2\pi}{3}\right)\right] \quad (16)$$

[1]Fitzgerald and Kingsley, Electric Machinery, McGraw Hill, 1952.

Let $C_{nk} = \frac{4N}{\pi} \frac{I_n K_{pk} K_{dk}}{k}$

Trigonometric expansion of the equations gives:

$$F(\gamma,t)_a = \frac{C_{nk}}{2}\left\{\cos[k\gamma - n\omega t + \nu] - \cos[k\gamma + n\omega t - \nu]\right\} \qquad (17)$$

$$F(\gamma,t)_b = \frac{C_{nk}}{2}\left\{\cos\left[k\gamma - n\omega t + \nu - 2\pi\left(\tfrac{k-n}{3}\right)\right] - \cos\left[k\gamma + n\omega t - \nu - 2\pi\left(\tfrac{k+n}{3}\right)\right]\right\} \qquad (18)$$

$$F(\gamma,t)_c = \frac{C_{nk}}{2}\left\{\cos\left[k\gamma - n\omega t + \nu + 2\pi\left(\tfrac{k-n}{3}\right)\right] - \cos\left[k\gamma + n\omega t - \nu - 2\pi\left(\tfrac{k+n}{3}\right)\right]\right\} \qquad (19)$$

Each equation represents two mmf waves sinusoidally distributed in space and moving in opposite directions at $\frac{n\omega}{k}$ radians per second.

The three phase mmfs are added to obtain the resultant mmf wave. The combination of terms containing $(k\gamma - n\omega t)$ will give:

$$3/2 \cos(k\gamma - n\omega t + \nu)$$

when $\frac{k-n}{3}$ = an integer or zero.

The combination of terms containing $(k\gamma + n\omega t)$ will give:

$$3/2 \cos(k\gamma + n\omega t - \nu)$$

when $\frac{k+n}{3}$ = an integer or zero. This is true since, for these conditions, the components are in phase and add directly. When $\frac{k \pm n}{3} \neq$ an integer or zero, the terms are $\frac{2\pi}{3}$ radians apart and add to zero.

Thus the resultant mmf expression is:

$$F(\gamma,t)=3/2\Big\{C_{11}\cos(\gamma-\omega t+\alpha)-C_{51}\cos(5\gamma+\omega t-\alpha)$$
$$+C_{71}\cos(7\gamma-\omega t+\alpha)+C_{33}\cos(3\gamma-3\omega t+\beta)$$
$$-C_{33}\cos(3\gamma+3\omega t-\beta)-C_{15}\cos(\gamma+5\omega t-\delta)$$
$$+C_{55}\cos(5\gamma-5\omega t+\delta)-C_{75}\cos(7\gamma+5\omega t-\delta)$$
$$+C_{17}\cos(\gamma-7\omega t+\xi)-C_{57}\cos(5\gamma+7\omega t-\xi)$$
$$+C_{77}\cos(7\gamma-\omega t+\xi)+\cdots\cdots\Big\} \qquad (20)$$

As pointed out previously, the mmf waves move at $\frac{n\omega}{k}$ radians per second. Converted to revolutions per minute it becomes $\frac{2f60n}{Pk}$ where f is the fundamental frequency and P is the number of poles. The synchronous speeds of the fundamental component of mmf is $\frac{120f}{P}$ Then the synchronous speed S which the various n and k harmonics tend to produce are shown in Table I. The plus signs denote rotation in the forward direction. The negative signs denote rotation in the opposite direction.

TABLE I

Speeds of the Various Harmonics in Terms of the Fundamental Speed S.

n \ k	1	3	5	7
1	+S	0	-S/5	+S/7
3	0	±S	0	0
5	-5S	0	+S	-5S/7
7	+7S	0	-7S/5	+S

Equation (20) shows that the k^{th} space harmonic of any current harmonic is reduced by the factor $\frac{K_{pk} K_{dk}}{k}$ Values of K_{pk} and K_{dk} are given in Tables II and III. By choosing the proper pitch and number of slots per pole the effect of space harmonics higher than the third may be neglected.[1] By disconnecting the neutral, the third harmonic and multiples of the third harmonic space fields are eliminated for wye windings.

TABLE II

Pitch Factor

Space Harmonic k	Pitch 7/9	4/5	5/6
1	.940	.951	.966
3	.500	.588	.707
5	.174	.000	.259
7	.644	.588	.259

TABLE III

Distribution Factor

Space Harmonic k	Slots per Pole 6	9	12
1	.966	.960	.958
3	.707	.667	.653
5	.259	.218	.205
7	.259	.177	.157

[1]Fitzgerald and Kingsley, Electric Machinery, McGraw Hill, 1952, pp 196-201.

3. Theoretical Analysis of Torque.

For the fundamental space wave the maximum torque[1] of an induction motor is:

$$|T_{max}|_1 = \frac{P}{8\pi f_1} \cdot \frac{E_1^2}{R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}}$$

where E_1, is the voltage applied to the motor, R_1, is the stator ohmic resistance, X_1, is the stator reactance and X_2 is the rotor reactance referred to the stator. This equation will apply to any time harmonic of order n, provided there is a rotating flux in the stator and an induced current in the rotor, both with the same frequency. The torque for the n^{th} voltage harmonic becomes:

$$|T_{max}|_n = \frac{P}{8\pi f_n} \frac{E_{1n}^2}{R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)_n^2}}$$

The relation between the n^{th} harmonic voltage and the fundamental voltage for a square wave is:

$$|E_{1n}| = \left|\frac{E_1}{n}\right|$$

The ratio of the n^{th} voltage harmonic torque to the fundamental torque is:

$$\frac{|T_{max}|_n}{|T_{max}|_1} = \frac{P\, 8\pi f_1}{P\, 8\pi f_n} \cdot \frac{E_{1n}^2}{E_1^2} \frac{R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}}{R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)_n^2}}$$

[1] Lawrence and Richards, Principles of Alternating Current Machinery, Fourth Edition, McGraw Hill, 1953, p 406.

$$\frac{|T_{max}|_n}{|T_{max}|_1} = \frac{f_1}{f_n} \frac{(E_{1n})^2}{(E_1)^2} \frac{R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}}{R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)_n^2}}$$

$$= \frac{f_1}{n f_1} \frac{(E_1/n)^2}{(E_1)^2} \frac{R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}}{R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)_n^2}}$$

$$= \frac{1}{n^3} \frac{R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}}{R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)_n^2}}$$

assume:

$$(X_1+X_2)_n > (X_1+X_2)$$

and:

$$R_{1n} \geqq R_1$$

then:

$$(X_1+X_2)_n^2 > (X_1+X_2)^2$$

$$\sqrt{R_{1n}^2 + (X_1+X_2)_n^2} > \sqrt{R_1^2 + (X_1+X_2)^2}$$

$$R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)_n^2} > R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}$$

$$\frac{R_1 + \sqrt{R_1^2 + (X_1+X_2)^2}}{R_{1n} + \sqrt{R_{1n}^2 + (X_1+X_2)^2}} < 1$$

$$\frac{|T_{max}|_n}{|T_{max}|_1} \leqq \frac{1}{n^3}$$

Therefore the torque caused by the fifth and higher order harmonics may be neglected.

4. Equipment.

A harmonic generator set consisting of a direct current motor and five alternating current generators coupled in tandem was used to produce the desired voltage wave. At rated speed of 3600 revolutions per minute the generator produced 60, 120, 180, 300 and 420 cycle, three phase alternating voltages. The stators of the harmonic generators can be rotated so that the harmonic voltages may be shifted in phase. The three phases of the sixty cycle generator were wye connected. The 180 cycle generator produced phase voltages that were displaced 120 degrees from each other. Therefore it was necessary to use phase shifters to put these voltages in phase with one another. In order to get the proper voltage, a delta-wye connected transformer was used to boost the voltage input to the phase shifters. (See Fig. 5) Transformers were used to boost the 300 cycle voltage. Two phases of the 300 cycle generator were reversed to obtain a negative sequence of voltages.

The phases of each harmonic generator were connected to a switchboard where they could be placed in series or removed from the circuit. The switchboard allowed any combination of generators to be placed in series.

The motor used in the experimental work was an ELECTRO DYNAMIC one horse-power, four pole, squirrel cage induction motor, rated at 220 volts, 60 cycles, 40°C rise, continuous duty. The windings are inter-connected externally and the coils have a 7/9 pitch with three slots per pole per phase. This motor was designed for student use in the study of winding connections. The large number of external connections gives a larger winding resistance than would be expected in a commercial motor.

The motor was coupled to a KIMBALL ELECTRIC 2KW, 125 volt direct current generator with a quick-break coupling. The generator was separately excited.

Two GENERAL RADIO wave analyzers (MOD 736-A) were used to measure the relative magnitudes of the harmonic voltages and currents.

A DUMONT dual beam cathode ray oscilloscope (Type 279) was used to observe the phase relationships between harmonic voltages so that a square voltage wave could be maintained for all loads.

5. General Procedure

All wattmeters, voltmeters, and ammeters were calibrated using 0.1% secondary standards. Also a check of the instruments was made at different frequencies to determine frequency erros. The speed of the harmonic generator was kept at 3600 revolutions per minute by using a stroboscope connected to the local commercial power supply. A motor field rheostat was used to control the speed of the harmonic generator set.

The torque output of the induction motor was determined by measuring the power output of the direct current generator loaded by a resistor bank. The direct current generator field current was held constant. The stray power was then determined for this field current. The brush and copper losses in the armature were determined by applying different voltages to the armature with the armature blocked. Graphs of these losses and the voltage versus current are shown in Fig. 6.

The wave shapes of the motor voltage and current were determined by the wave analyzer and the cathode ray oscilloscope. The phase angles of the harmonic components were determined from the wave form displayed on the oscilloscope. The magnitudes of the harmonic components were determined by the wave analyzer. To obtain the current wave forms, identical resistors were placed in each line and the wave analyzer and oscilloscope were connected across these resistors.

With the neutral connected, the third harmonic current was measured by placing an ammeter in the neutral line. A check of the root-mean-square voltage and current was obtained from the voltmeter and ammeter readings.

The speed of the induction motor was determined by use of a chronometric tachometer. The power input to the motor was measured by placing

a wattmeter in each phase.

6. Experimental Results

A test run was made on the motor using an approximation of the square voltage wave (Fig. 7) with the neutral not connected. Since the motor was rated at 127 volts per phase the root-mean-square value of this wave was adjusted to this value. This was accomplished by setting $E_1=117$, $E_3=39$, $E_5=23.5$, and $E_7=16.7$ volts root-mean-square. Thus the root-mean-square voltage of the wave was:

$$E_{rms}=\sqrt{E_1^2+E_3^2+E_5^2+E_7^2} = 127 \text{ volts}$$

The wave shape and root-mean-square magnitude were held invariant during loading. For this run there was negligible third harmonic current. The fifth and seventh harmonic currents remained almost constant at 0.4 and 0.2 amperes respectively. Runs were then made at 117 volts and 127 volts per phase with a sixty cycle sinusoidal voltage. A comparison of these three runs is shown in Fig. 8. It is seen that the effect of the harmonics was negligible, i.e., only the 117 volt fundamental voltage is effective in producing torque.

Due to the poor voltage-regulation characteristics of the harmonic generator set, the line current was limited to five amperes maximum, hence the speed-torque curves obtained are limited to values between 1500 and 1800 revolutions per minute.

The neutral was then connected to determine the effect of the third harmonic current on the motor torque and efficiency. First a run was made with only the fundamental and third harmonic voltages. This gave a root-mean-square voltage of 123 volts. Next a run was made with the fifth and seventh harmonics added. The two runs produced the same torques and efficiencies. The results are shown in Fig. 9.

With this neutral connection, a great torque was produced than with the 117 volt fundamental alone, also there was a marked decrease in efficiency. (Fig. 9). However at a speed of about 1600 revolutions per minute the fundamental plus third harmonic torque curve crosses under that of the 117 volt fundamental torque curve.

The cause of this reduction in torque is shown in Fig. 10. Both curves are a plot of fundamental current versus speed. In one case only the fundamental voltage of 117 volts is applied to the motor, while in the other a fundamental voltage of 117 volts plus a third harmonic voltage of 39 volts is applied with the neutral connected. Comparison of these two curves shows a decided decrease in fundamental current at about 1600 revolutions per minute when the third harmonic is present. This decrease in fundamental current causes a decrease in the fundamental torque and is the reason for the cross-over shown in Fig. 9.

To test the analytical result expressed in Equation (20) that the third harmonic produces equal magnitude fields rotating in opposite directions, the motor was run on the third harmonic voltage alone. Since the third harmonic produced no starting torque, it was necessary to start the motor with the fundamental and then to switch off the fundamental leaving the motor running on the third harmonic voltage. It was possible to keep the motor running in either direction.

With a third harmonic voltage of 43 volts, the no-load speed was 1764 revolutions per minute. This indicated that the synchronous speed of the motor for the third harmonic is 1800 revolutions per minute. By touching the shaft lightly the motor could be stopped.

7. Conclusions.

In this investigation, the operating characteristics of a three phase squirrel cage induction motor run on a square wave voltage are compared to the operating characteristics when run on a sinusoidal voltage. Theoretical and experimental analyses were made. The conclusions from the theoretical and experimental analyses for the same conditions agree remarkably well.

By connecting the stator windings in wye without a neutral, the third harmonic and multiples of the third harmonic stator current are eliminated. With this connection the torque and efficiency for the motor are, for practical purposes, the same as that produced by the fundamental sine wave component of the square wave. This small change in efficiency indicates that the higher harmonics do not appreciably increase the core losses. There is no noticeable increase in the noise or vibration of the motor.

With the neutral disconnected, the maximum value of the square wave phase voltage (E_{sw}) will be related to the root-mean-square value of the sine wave phase voltages as follows:

$$\frac{4}{\pi} E_{sw} = E_{1(max)} = E_{1(rms)}\sqrt{2}$$

so that:

$$E_{sw} = \frac{\sqrt{2}\,\pi}{4} E_{1(rms)} = 1.11\ E_{1(rms)}$$

That is, to obtain the same performance, a motor rated at 127 volts per phase should be operated on a square wave phase voltage of 141 volts. Since the effect of the higher harmonics is negligible, the torque and

efficiency of the motor can be obtained by normal sinusoidal analysis using the fundamental component of the square wave voltage.

If the neutral is connected, the third harmonic and multiples of the third harmonic current will be present. The third harmonic current causes a decided increase in noise and vibration as slip increases. The third harmonic current produces a small increase in torque, however the copper losses of the motor are greatly increased. This increase in losses would require a larger motor for a given rating, therefore operation with the neutral connected is not desirable.

There are several items which were not investigated because of the lack of time or equipment that might require future investigation:

(a) The starting torque characteristics.

(b) The effect of the harmonic of the voltage wave which corresponds to the mmf harmonic caused by the tooth and slot combinations.

(c) The operation of a motor on the actual voltage output from a static inverter.

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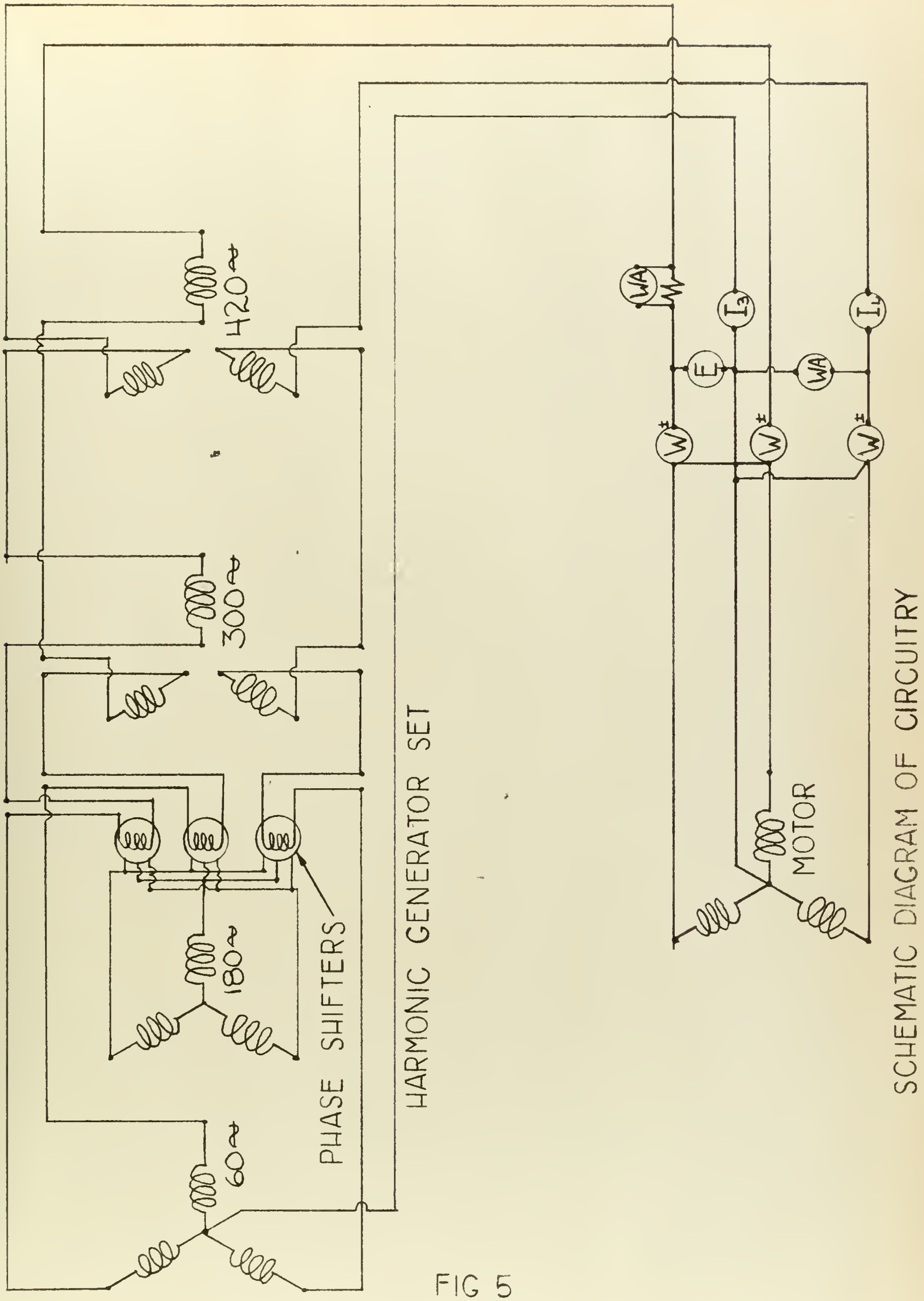

420~
300~
180~
60~
PHASE SHIFTERS
HARMONIC GENERATOR SET
WA
E
WA
W±
W±
W±
MOTOR
SCHEMATIC DIAGRAM OF CIRCUITRY

FIG 5

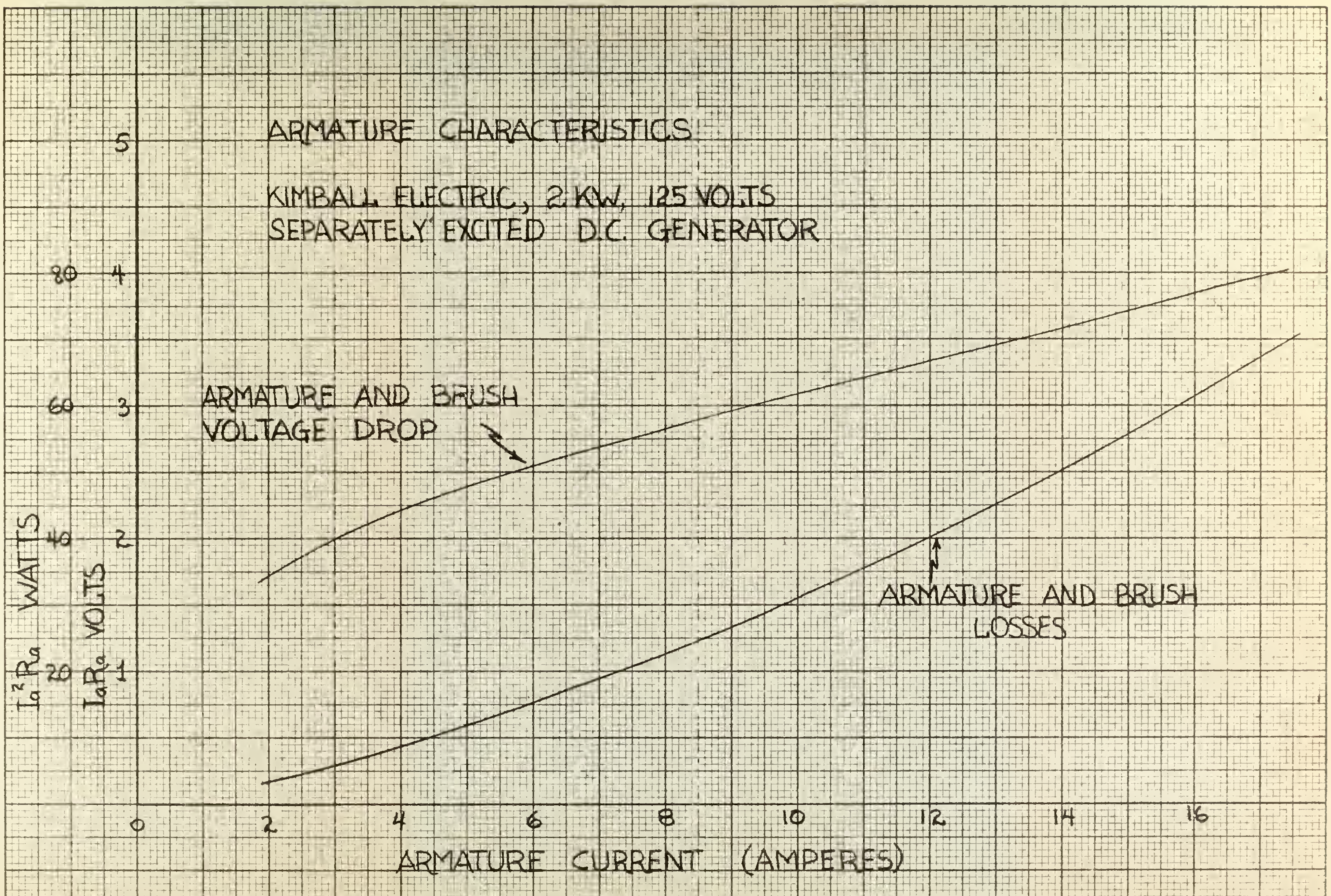

ARMATURE CHARACTERISTICS
KIMBALL ELECTRIC, 2 KW, 125 VOLTS
SEPARATELY EXCITED D.C. GENERATOR
ARMATURE AND BRUSH
VOLTAGE DROP
ARMATURE AND BRUSH
LOSSES
$I_a^2 R_a$ WATTS
$I_a R_a$ VOLTS
80
60
40
20
5
4
3
2
1
0
2
4
6
8
10
12
14
16
ARMATURE CURRENT (AMPERES)

FIGURE 6

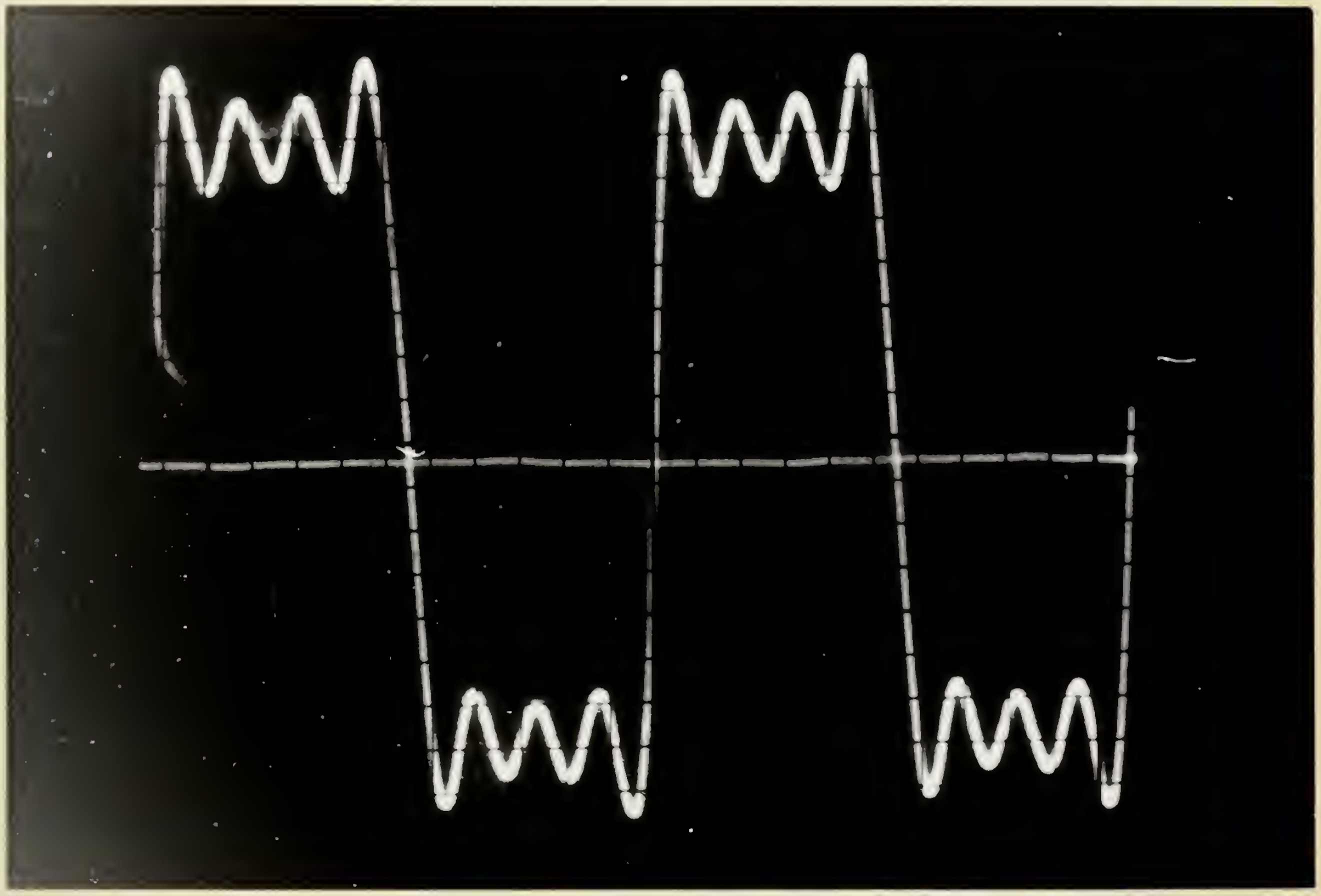


Fig. 7 Voltage Wave Shape

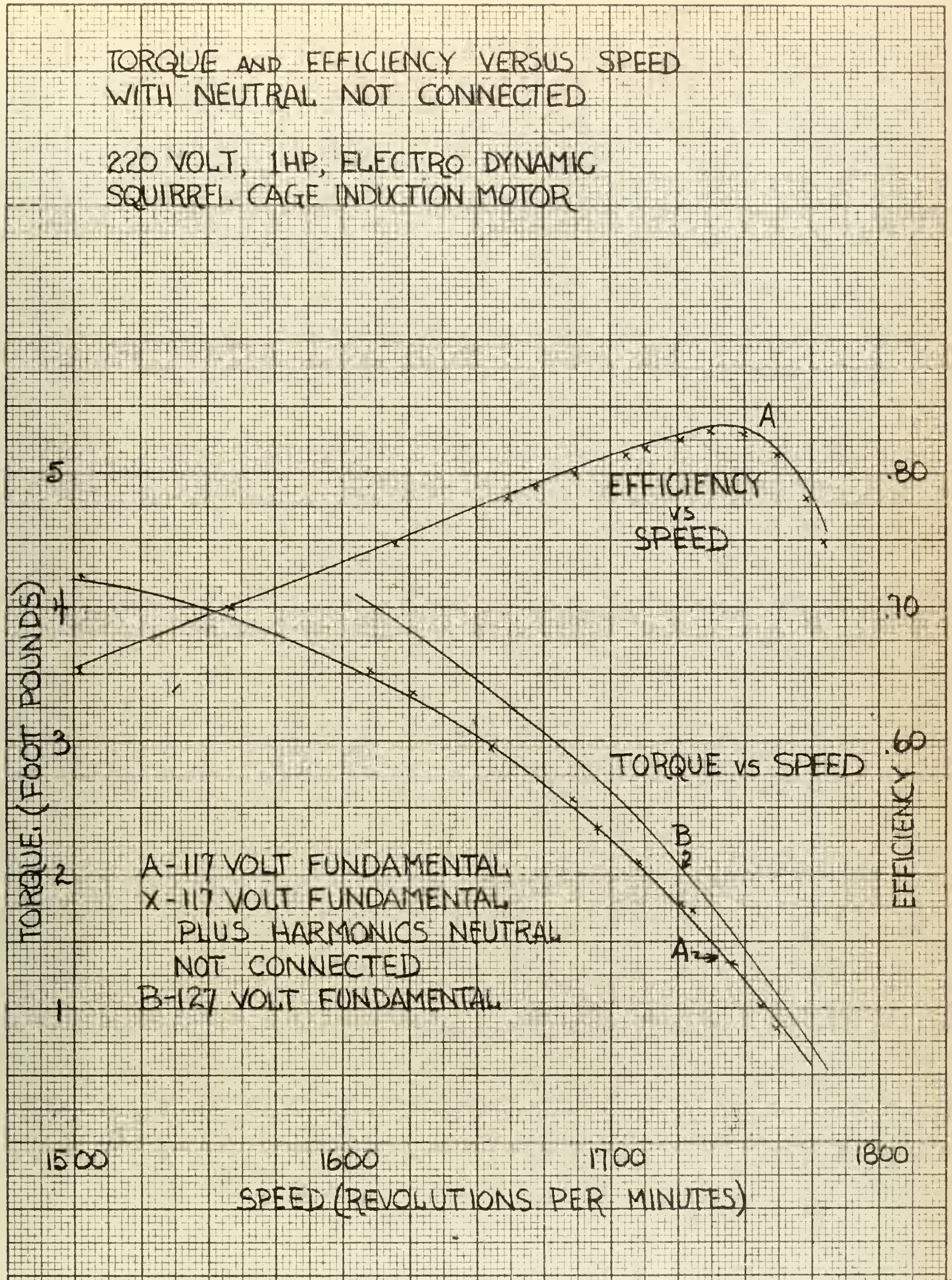
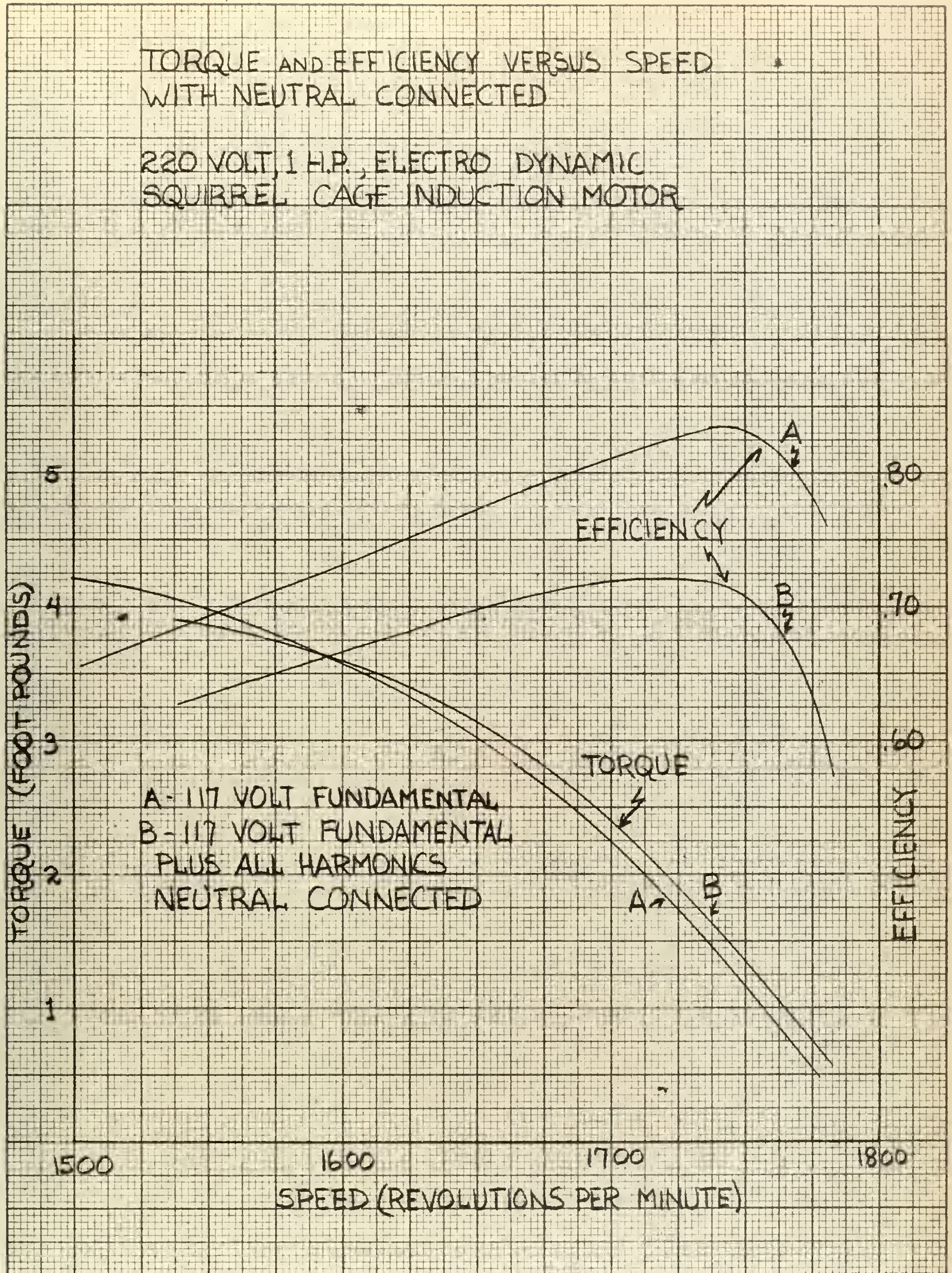

TORQUE AND EFFICIENCY VERSUS SPEED
WITH NEUTRAL NOT CONNECTED
220 VOLT, 1HP, ELECTRO DYNAMIC
SQUIRREL CAGE INDUCTION MOTOR
A
EFFICIENCY
vs
SPEED
TORQUE vs SPEED
B
2
A
2
A - 117 VOLT FUNDAMENTAL
X - 117 VOLT FUNDAMENTAL
PLUS HARMONICS NEUTRAL
NOT CONNECTED
B-127 VOLT FUNDAMENTAL
5
4
3
2
1
.80
.70
.60
TORQUE (FOOT POUNDS)
EFFICIENCY
1500
1600
1700
1800
SPEED (REVOLUTIONS PER MINUTES)

FIGURE 8



TORQUE AND EFFICIENCY VERSUS SPEED
WITH NEUTRAL CONNECTED
220 VOLT, 1 H.P., ELECTRO DYNAMIC
SQUIRREL CAGE INDUCTION MOTOR
A
EFFICIENCY
B
TORQUE
A - 117 VOLT FUNDAMENTAL
B - 117 VOLT FUNDAMENTAL
PLUS ALL HARMONICS
NEUTRAL CONNECTED
A
B
TORQUE (FOOT POUNDS)
EFFICIENCY
5
4
3
2
1
.80
.70
.60
1500
1600
1700
1800
SPEED (REVOLUTIONS PER MINUTE)

FIGURE 9

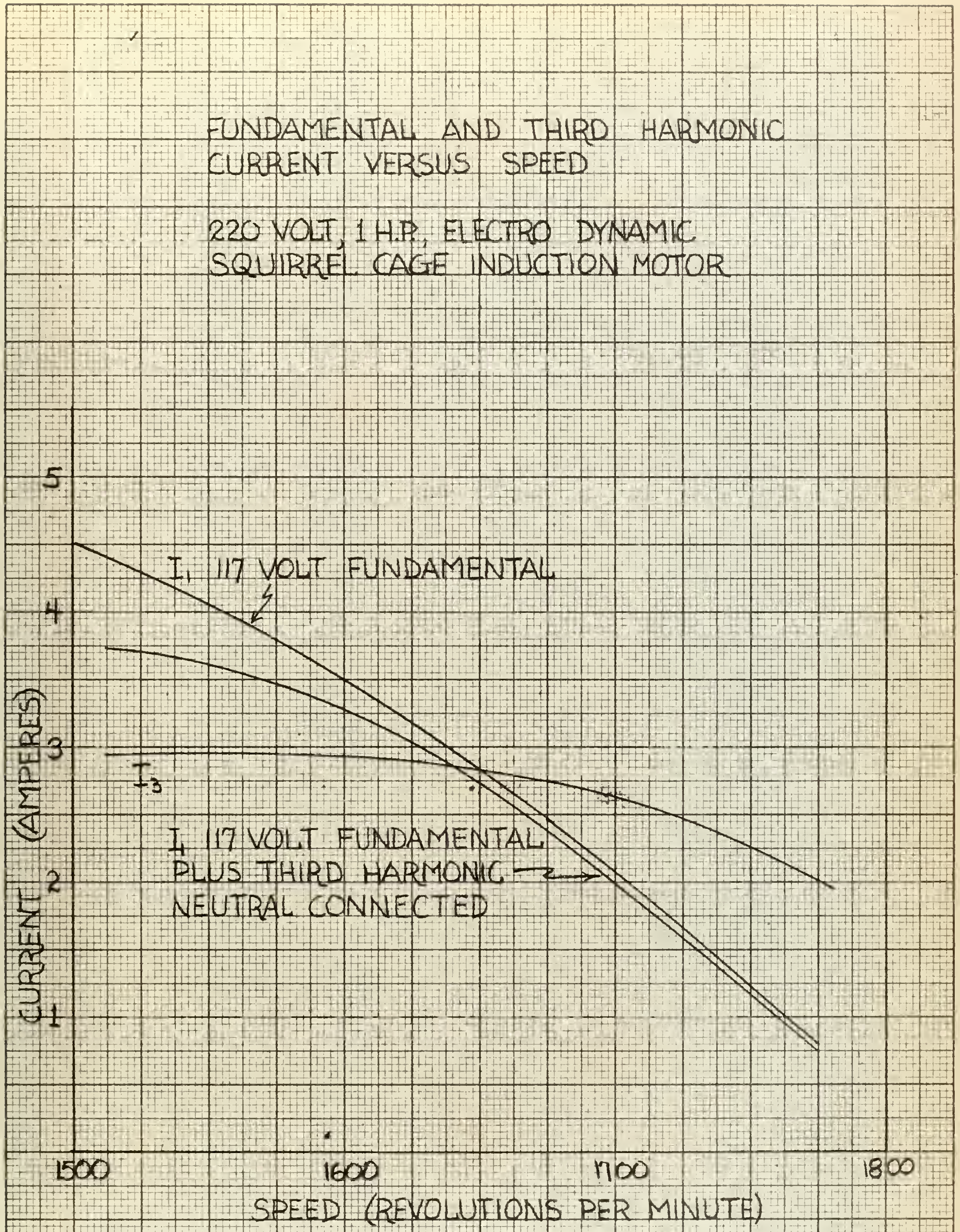

FUNDAMENTAL AND THIRD HARMONIC
CURRENT VERSUS SPEED
220 VOLT, 1 H.P., ELECTRO DYNAMIC
SQUIRREL CAGE INDUCTION MOTOR
I1 117 VOLT FUNDAMENTAL
I3
I4 117 VOLT FUNDAMENTAL
PLUS THIRD HARMONIC
NEUTRAL CONNECTED
CURRENT (AMPERES)
1
2
3
4
5
1500
1600
1700
1800
SPEED (REVOLUTIONS PER MINUTE)

FIGURE 10

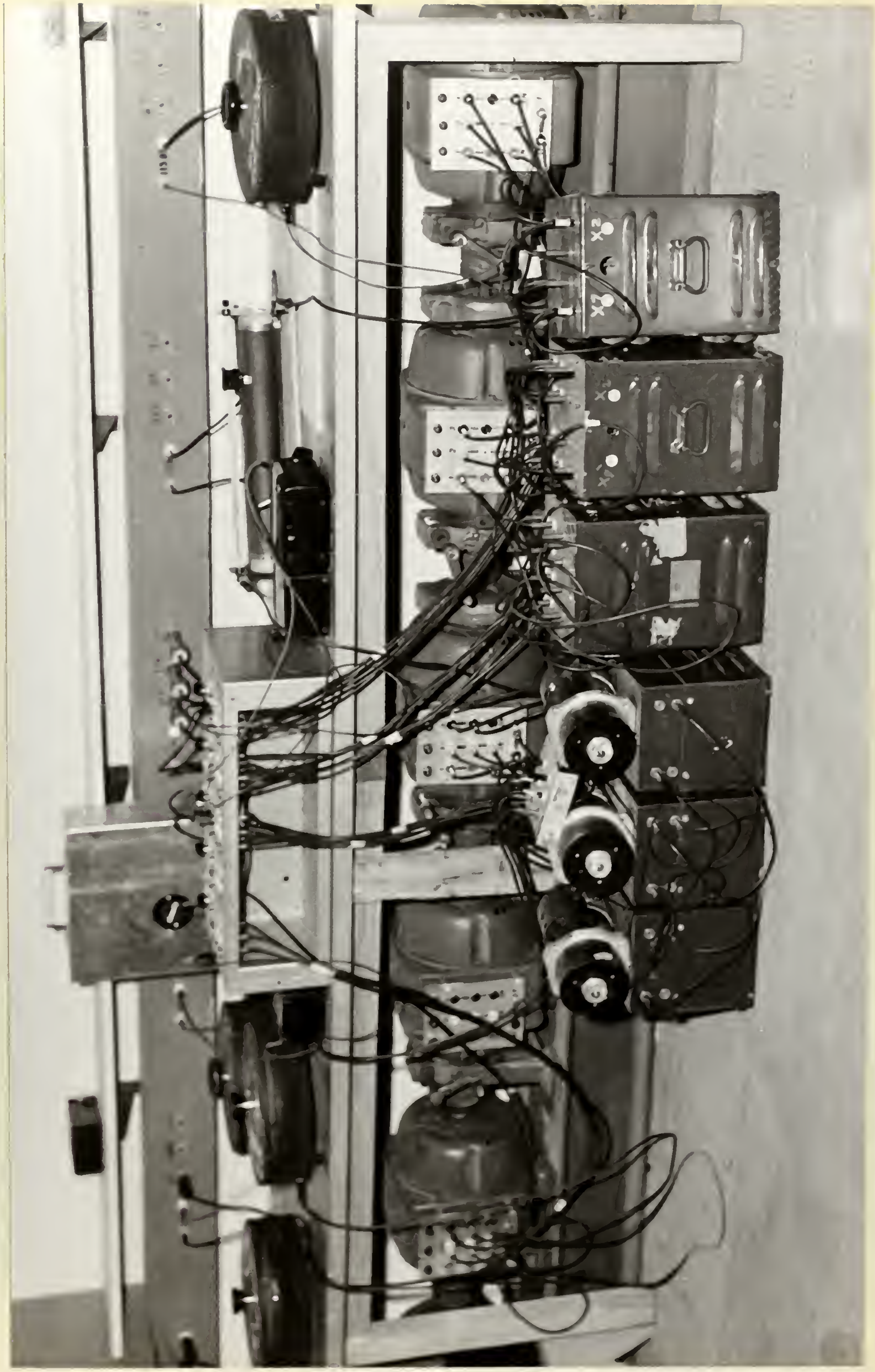


Fig.11 Harmonic Generator Set

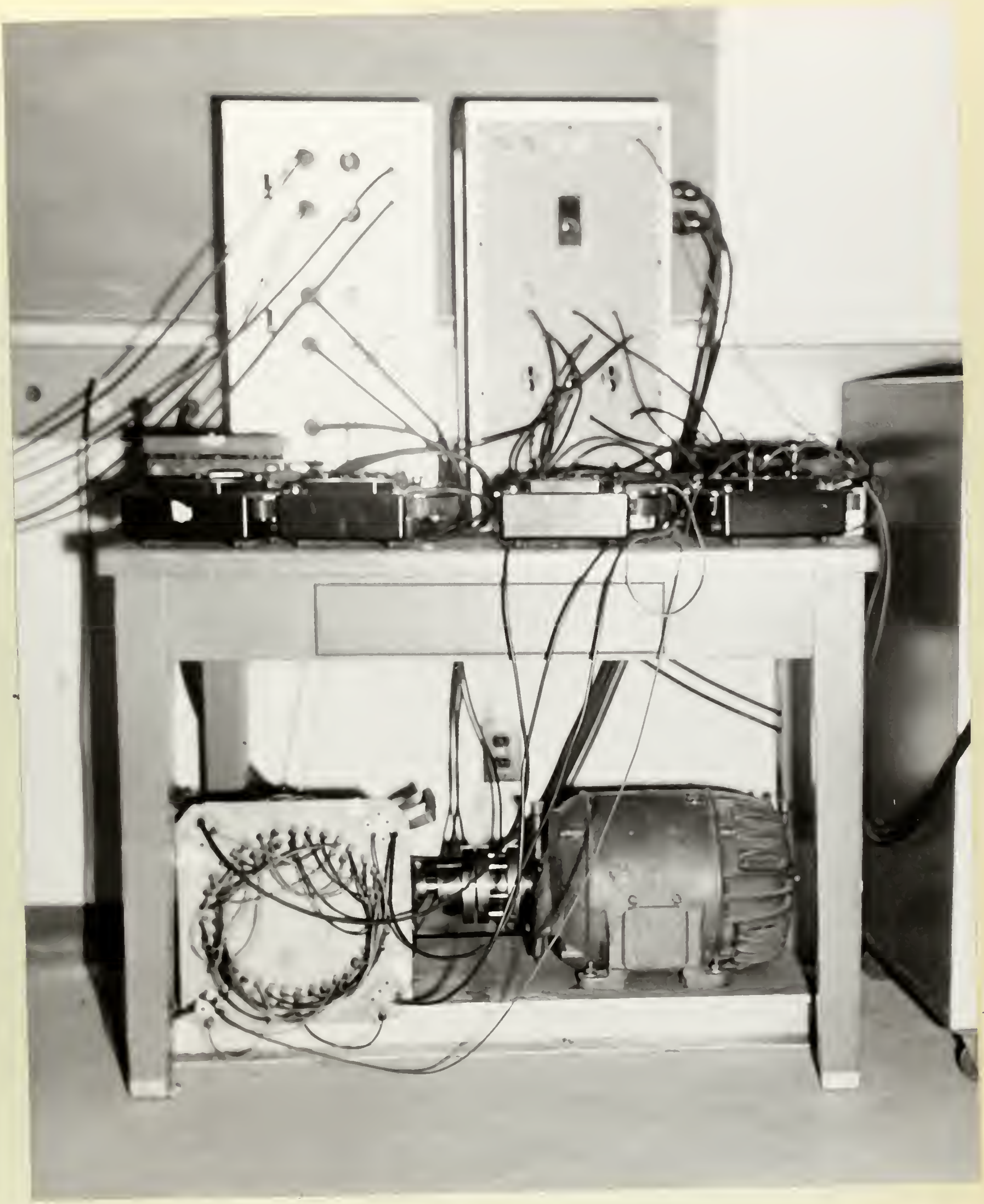


Fig. 12 Induction motor and D-C Generator